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Persistent current in a one-dimensional ring induced by an electromagnetic potential

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Abstract. In this paper, we study the persistent current in a normal conductor ring coupled to a reservoir with an ideal lead and investigate the effect of an electromagnetic potential on free electrons in a normal-conductor one-dimensional ring. The electrostatic potential on the ring modulates the amplitude, phase and frequency of the Aharonov–Bohm flux current. We discuss the effect of reflection on both sides of the gate electrode on the persistent current in the presence of an electromagnetic potential. The oscillation amplitude of the persistent current and the density of states decrease as the electrostatic potential increases at constant magnetic flux.

1. Introduction

Quantum interference phenomena in mesoscopic rings induced by magnetic flux have been the subject of recent experimental and theoretical studies, including a number of investigations of quantum oscillations in pure one-dimensional rings with one or two ideal leads threaded by magnetic flux [1–3]. It was recently pointed out that interference between the two paths of the ring may also be produced by applying an electrostatic potential [4, 5], and electrostatic Aharonov–Bohm (AB) conductance oscillations have been shown to be very different from cases involving magnetostatic AB oscillations. In recent studies, we investigate the AB type of conductance oscillations in the presence of both a magnetic flux and an electrostatic potential [6] and extend the calculation to evaluate the persistent current in the ring [7] and to provide a complete treatment of the electrostatic AB conductance oscillations [8–10]. The persistent current subject to magnetic flux has been studied by many workers [11], but the influence of an electrostatic potential on the persistent current has not been investigated so far. In this paper, we report theoretical studies of persistent current subject to an electromagnetic potential. We extend our previous model [7] to include the effect of an abrupt potential on both sides of a gate electrode to which an electrostatic potential is applied. We calculate the density of states and the persistent current and discuss the differences between the two models. This paper is organized as follows. In section 2, we develop a method for calculating the persistent current in the ring and derive rigorous expressions for the current density and the density of states in the ring. In section 3, we calculate the density of states and persistent current in the ring, and investigate the effect of reflection. Conclusions are given in section 4.

2. General formulation of the persistent current

The model investigated here is shown in figure 1. A one-dimensional normal conductor ring is penetrated by a magnetic flux, and an electrostatic potential is applied to half the

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circumference of the ring. A reservoir is coupled to the ring with an ideal lead. In our system, phase-coherent transport is assumed; so the inelastic scattering length is larger than the circumference of the ring. The junction between the ring and the lead is described by a scattering matrix [1,2] which relates the amplitude of the outgoing waves to the incoming waves

$$\begin{bmatrix}\alpha'\\ \beta'\\ \gamma'\end{bmatrix} = \begin{bmatrix}-(a+b) & \sqrt{\eta} & \sqrt{\eta}\\ \sqrt{\eta} & a & b\\ \sqrt{\eta} & b & a\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} \tag{1}$$

$$a = \tfrac{1}{2}\left(\sqrt{1-2\eta}-1\right)$$

$$b = \tfrac{1}{2}\left(\sqrt{1-2\eta}+1\right) \tag{2}$$

where η is the coupling strength at the junction. When $\eta = 0.5$, electrons travel almost ballistically through the junction. In contrast, they suffer from strong reflection at the junction when $\eta \simeq 0$. Symmetry of the scattering matrix (equation (1)) holds when the magnetic flux is very weak. In the limit of weak scattering at the junction, the Hamiltonian in the ring is described by

$$H = \begin{cases}(p - eA/c)^2/2m & \text{(upper half of the ring)}\\ (p - eA/c)^2/2m + eV & \text{(lower half of the ring)}\end{cases} \tag{3}$$

where $A = \Phi/L$ is the vector potential, V is the electrostatic potential applied to half the ring, m is the effective mass of electrons, Φ is the magnetic flux and L is the circumference of the ring. For simplicity, we assume that the electrostatic potential is abrupt. However, even in complete formulation of the electrostatic AB oscillations, we must consider the potential difference between the inside and the outside of the gate electrode, especially when the electrostatic potential is high. In the upper half of the ring, the relationship between the amplitudes of the outgoing and the ingoing waves is described by

$$\begin{bmatrix}\delta\\ \beta\end{bmatrix} = \begin{bmatrix}t_A & 0\\ 0 & t_B\end{bmatrix}\begin{bmatrix}\beta'\\ \delta'\end{bmatrix} \tag{4}$$

$$t_A = \exp(\tfrac{1}{2}ikL - \tfrac{1}{2}i\theta) \tag{5}$$

$$t_B = \exp(\tfrac{1}{2}ikL + \tfrac{1}{2}i\theta) \tag{6}$$

where $k = \sqrt{2mE}/\hbar$ is the wavenumber of the electrons and $\theta = e\Phi/\hbar$ is the magnetic phase shift. In the lower half of the ring, the relationship between the amplitude of outgoing and incoming waves is described by the following three scattering matrices:

$$\begin{bmatrix}\delta'\\ \epsilon'\end{bmatrix} = \begin{bmatrix}t' & r\\ r' & t\end{bmatrix}\begin{bmatrix}\epsilon\\ \delta\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}\epsilon\\ \zeta'\end{bmatrix} = \begin{bmatrix}t_C & 0\\ 0 & t_D\end{bmatrix}\begin{bmatrix}\zeta\\ \epsilon'\end{bmatrix} \tag{8}$$

$$\begin{bmatrix}\zeta\\ \gamma\end{bmatrix} = \begin{bmatrix}t & r'\\ r & t'\end{bmatrix}\begin{bmatrix}\gamma'\\ \zeta'\end{bmatrix} \tag{9}$$

$$t = 2k_1/(k_1 + k_2) \tag{10}$$

$$t' = 2k_2/(k_1 + k_2) \tag{11}$$

$$r = (k_1 - k_2)/(k_1 + k_2) \tag{12}$$

$$r' = (k_2 - k_1)/(k_1 + k_2) \tag{13}$$

$$k_1 = \sqrt{2mE}/\hbar \tag{14}$$

$$k_2 = \sqrt{2m(E + eV)}/\hbar \tag{15}$$

$$t_C = \exp(\tfrac{1}{2}\mathrm{i}kL + \tfrac{1}{2}\mathrm{i}\theta + \mathrm{i}\phi) \tag{16}$$

$$t_D = \exp(\tfrac{1}{2}\mathrm{i}kL - \tfrac{1}{2}\mathrm{i}\theta + \mathrm{i}\phi) \tag{17}$$

$$\theta = e\Phi/\hbar \tag{18}$$

$$\phi = \sqrt{2m}\left(\sqrt{E + eV} - \sqrt{E}\right)L/(2\hbar). \tag{19}$$

Using equations (4)–(19), we find the following relationship between the amplitudes at the entrance and exit points:

$$\begin{bmatrix} \beta' \\ \beta \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} \gamma \\ \gamma' \end{bmatrix} \tag{20}$$

$$P_{11} = (t_D^{-1} - r'^2 t_C)/tt't_A \tag{21}$$

$$P_{12} = (-rt_D^{-1} - r't_{CC})/tt't_A \tag{22}$$

$$P_{21} = (rt_B t_D^{-1} + r't_B t_C)/tt' \tag{23}$$

$$P_{22} = (t_B t_C - t_B r^2 t_D^{-1})tt'. \tag{24}$$

When reflection on both sides of the junction is neglected, equation (20) reduces to the following simple relationship:

$$\begin{bmatrix} \beta' \\ \beta \end{bmatrix} = \begin{bmatrix} \exp(-\mathrm{i}kL + \mathrm{i}\theta - \mathrm{i}\phi) & 0 \\ 0 & \exp(\mathrm{i}kL + \mathrm{i}\theta + \mathrm{i}\phi) \end{bmatrix} \begin{bmatrix} \gamma \\ \gamma' \end{bmatrix}. \tag{25}$$

The relationship in equations (25) and (20) is used in model 1 [7] and model 2. The circulating current in the ring is obtained by evaluating

$$J = e/h \int_0^\infty \mathrm{d}E\, f(E) D(E, \theta, \phi) \tag{26}$$

where $f(E)$ is the Fermi–Dirac distribution function $f(E) = (1 + \exp[(E - E_F)/kT])^{-1}$. At $T = 0$, we integrate over the energy up to the Fermi energy $E_F = \hbar^2(2n + 1)^2/(2mL^2)$, where n is integer. The numerical calculations are all done with $n = 2$. The function $D(E, \theta, \phi)$ is the current density divided by the velocity of electrons in the ring and is given by

$$D(E, \theta, \phi) = (|\beta'|^2 - |\beta|^2)/(|\alpha|^2). \tag{27}$$

The density of states in the ring is defined by

$$\mathrm{DOS}(E,\theta,\phi) = C(|\beta'|^2 + |\beta|^2)/(|\alpha|^2) \qquad (C \text{ is a constant}) \tag{28}$$

where the ratio of the amplitude (α) of the incident waves from the ideal lead to the amplitude (β') of the outgoing waves from the junction or the amplitude (β) of the incoming waves of the ring is given by

$$\beta'/\alpha = \sqrt{\eta}P_4[P_{11} + (a-b)(P_4 + P_{12})]/F(E,\theta,\phi) \tag{29}$$

$$\beta/\alpha = \sqrt{\eta}P_4[P_4 + P_{12} + (a-b)P_{22}]/F(E,\theta,\phi) \tag{30}$$

$$P_4 = P_{11}P_{22} - P_{12}P_{21} \tag{31}$$

$$F(E,\theta,\phi) = (bP_{22} - P_4)(bP_4 - aP_{12} - P_{11}) - (aP_{22} + P_{21})(aP_4 - bP_{12}). \tag{32}$$

The current density, density of states, and persistent current are calculated using equation (1) and equations (26)–(32). Within the ballistic-transport limit on both sides of the gate electrode, our model reduces to model 1 [7] which calculates the current density, density of states and persistent current using equation (1) and equations (25)–(28).

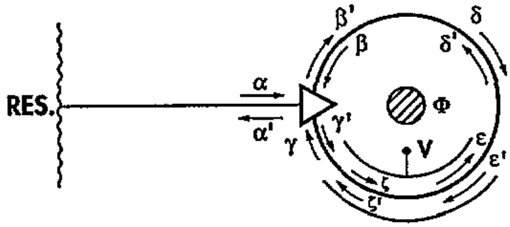

Figure 1. One-dimensional ring coupled to a reservoir. Magnetic flux threads the centre of the ring and an electrostatic potential is applied to half the ring. The model considers the effect of reflection at the gate electrode.

3. Calculation of persistent current

In this section, we calculate the current density and persistent current using model 1 and model 2, which were described in section 2. In model 2, the wavenumbers inside and outside the gate electrode are given by $\sqrt{2m(E_F + eV)}/\hbar$ and $\sqrt{2mE_F}/\hbar$, respectively at the Fermi energy E_F. Model 1 assumes that both wavenumbers are the same. The reflection coefficient at the gate electrode is expressed in terms of E_F and eV and is given by

$$r' = \left(\sqrt{1 + eV/E_F} - 1\right) \Big/ \left(\sqrt{1 + eV/E_F} + 1\right). \tag{33}$$

When $eV/E_F \ll 1$, the wavenumber of electrons varies slowly at the boundary of the gate electrode. In this case, model 2 reduces to model 1. If $eV/E_F > 1$, the effect of reflection at the edges of the gate electrode becomes essential. These features are shown in figure 2. The amplitude of the current density D versus ϕ gradually decreases as the

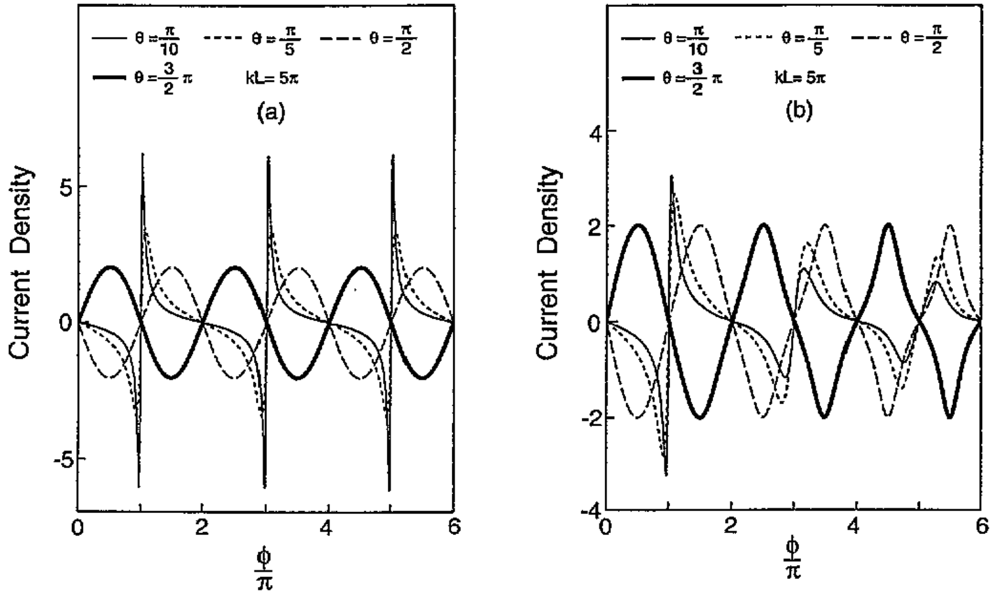

Figure 2. (*a*) The current density as a function of ϕ for model 1. The magnetic flux θ is used as a parameter ($k_F L = 5\pi$). (*b*) The current density versus ϕ for model 2 ($k_F L = 5\pi$).

potential increases (at constant θ) owing to reflection on both sides of the electrode (see figure 2(*b*)). In contrast, the current density oscillates exactly with period 2π as a function of ϕ in model 1 as shown in figure 2(*a*).

On the other hand, the current density as a function of magnetic flux (current density versus θ with ϕ constant) oscillates with period 2π. The oscillation amplitude does not decrease irrespective of ϕ and, as a result, both models are very similar. This is because the Fermi energy and electrostatic potential are fixed.

Next, we shall discuss the properties of persistent current in the ring. The amplitude, phase and frequency of the persistent current as a function of magnetic flux (at constant V) are greatly modulated by the applied electrostatic potential. The persistent current oscillates smoothly as a function of magnetic flux since model 1 assumes ballistic transport in the ring (see figure 3(*a*)). When $\phi \simeq (n + \frac{1}{2})\pi$, $h/2e$ oscillations appear and the phase of the persistent current shifts by π at $\theta \simeq (2n + 1)\pi$. In contrast, the persistent current in model 2 fluctuates slightly and the phase is slightly shifted from the phase ϕ calculated using model 1 as shown in figure 3(*b*). This fluctuation and phase shift in persistent current becomes more significant as V increases, owing to carrier reflection on both sides of the electrode. This tendency becomes stronger as the electrostatic potential rises. Let us now investigate the persistent current as a function of electrostatic potential (at constant θ). In the absence of magnetic flux, or more generally when $\theta = n\pi$, there is no circulating current in the ring irrespective of the electrostatic potential. When $\theta \not\simeq n\pi$, however, the persistent current oscillates as a function of ϕ. The amplitude and phase are modulated by applying magnetic flux. The persistent current oscillates periodically without fluctuation as a function of electrostatic potential in model 1 irrespective of θ (figure 4(*a*)). As expected from the current density versus ϕ relationship (at constant θ) when ϕ is low, both models are similar. As the electrostatic potential increases, the oscillation amplitude gradually decreases and fluctuates greatly owing to strong reflection at the edges of the gate electrode in model 2, as shown in figure 4(*b*).

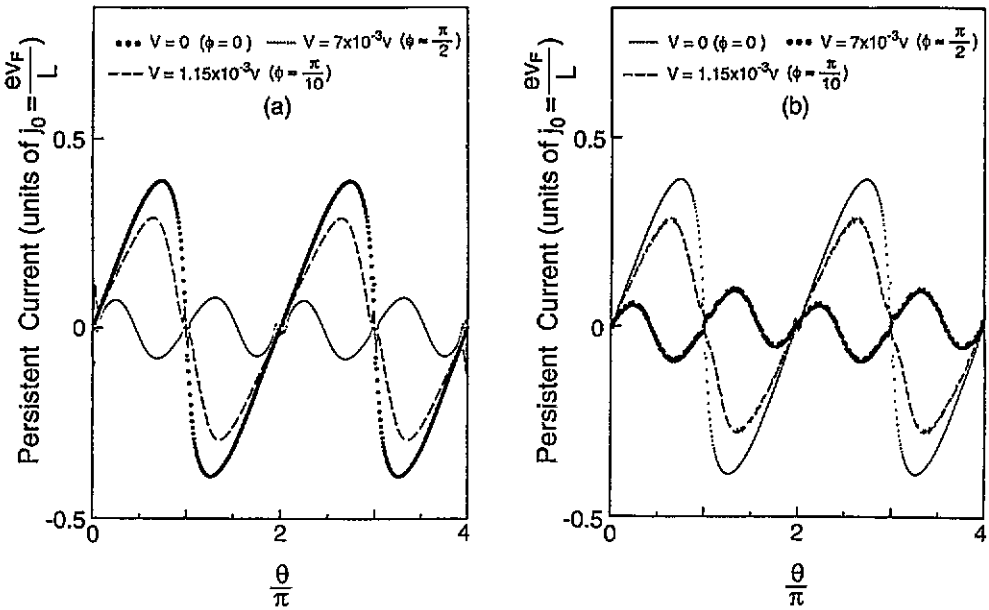

Figure 3. (*a*) Persistent current as a function of magnetic flux θ for model 1 (V is constant). V is used as a parameter. (*b*) Persistent current versus θ for model 2 (V is constant). V_F is the Fermi velocity of the ring.

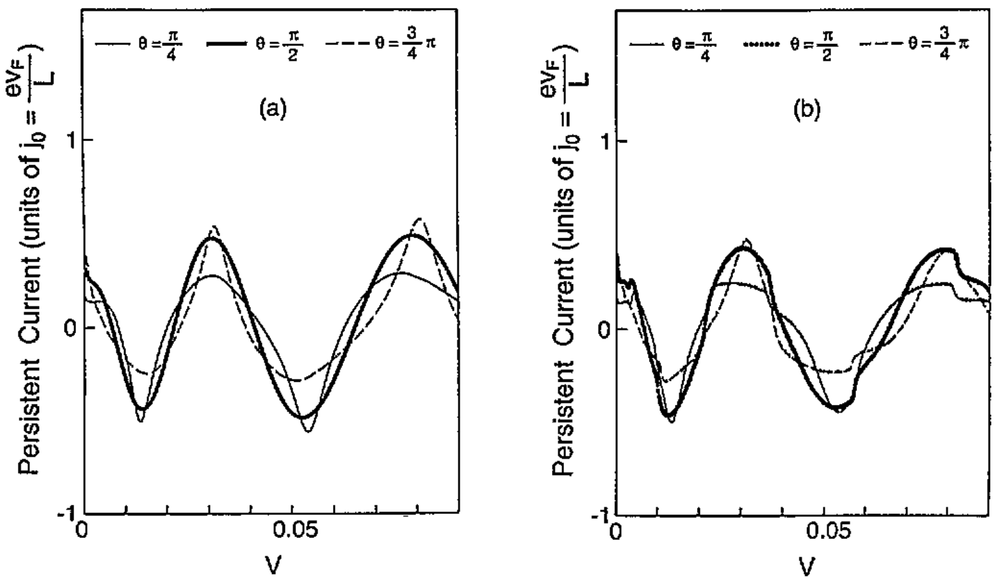

Figure 4. (*a*) Persistent current as a function of V for model 1 (θ is a constant). The magnetic flux is used as a parameter. (*b*) Persistent current versus V for model 2 (θ is constant).

4. Conclusion

We have presented a nearly complete formulation of persistent current in a one-dimensional ring subject to an electromagnetic potential. We have computed the current density and persistent current in the ring as functions of magnetic flux and electrostatic potential. It has been demonstrated that the oscillation amplitude, the phase and the frequency of oscillations are greatly influenced by the detailed balance of the magnetic flux and electrostatic potential and $k_F L$. We have investigated the effect of reflection at the edges of the gate electrode and discussed this effect by comparison with our previous model (neglecting reflection at the gate electrode). Our present model has shown that the oscillation amplitudes of current density and persistent current decrease as the electrostatic potential increases.

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